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Discriminant in math

Use The Quadratic Formula Calculator to see Quadratic Formula and discriminant in Action! This calculator will solve any quadratic equation you type in (even if solutions are imaginary). To understand what the discriminant does, it's important that you have a good understanding of: Answer A parabola. Answer The solution can be thought of in two different ways. Algebraically, the solution occurs when y = 0. So the solution is where $$y =\red ax^2 + \blue bx + \color{green} c $$ becomes $$0 =\red ax^2 + \blue bx + \color{green} c $$. Graphically, since y = 0 is the x-axis, the solution is where the parabola intercepts the x-axis. (This only works for real solutions). In the picture below, the left parabola has 2 real solutions (red dots), the middle parabola has 1 real solution (red dot) and the right most parabola has no real solutions (yes, it does have imaginary ones). Answer It looks like .. a number. 5, 2, 0, -1 - each of these numbers is the discriminant for 4 different quadratic equations. Answer The discriminant is a number that can be calculated from any quadratic equation. A quadratic equation is an equation that can be written as $$ ax^2 + bx + c $$ (where $$a e 0 $$). Answer The discriminant for any quadratic equation of the form $$ y =\red a x^2 + \blue bx + \color {green} c $$ is found by the following formula and it provides critical information regarding the nature of the roots/solutions of any quadratic equation. $ \boxed{Formula} \\ \text{Discriminant } = \blue b^2 -4 \red a \color{green} c $ $ \boxed{Example} \\ \text{Equation :} y =\red 3 x^2 + \blue 9x + \color {green} 5 \\ \text{Discriminant } = \blue 9^2 -4 \cdot \red 3 \cdot \color{green} 5 \\ \text{Discriminant } = \boxed{ 6} $ Answer The discriminant tells us the following information about a quadratic equation: If the solution is a real number or an imaginary number. If the solution is rational or if it is irrational. If the solution is one unique number or two different numbers. Value of the discriminant Type and number of Solutions Example of graph $ b^2 - 4ac > 0 $ $ \text{Example :} \\ y = \red 3x^2 \blue{-6}x + \color{green} 2 \\ \text{Discriminant} \\ \blue{-6}^2 - 4 \cdot \red 3 \cdot \color{green} 2 \\ = \boxed{12} \\ $ If the discriminant is a positive number, then there are 2 real solutions. This means that the graph of the parabola interepts the x-axis at 2 distinct points . $ \text{Example :} \\ y = \red 3x^2 + \blue 4 x \color{green} {-4} \\ \text{Discriminant} \\ \blue 4^2 - 4 \cdot \red 3 \cdot \color{green} {-4} \\ = \boxed{64} \\ $ If the discriminant is positive and also a perfect square like 64, then there are 2 real rational solutions. $ \text{Example :} \\ y = \red 3x^2 \blue {-6} x + \color{green} 2 \\ \text{Discriminant} \\ \blue {-6}^2 - 4 \cdot \red 3 \cdot \color{green} 2 \\ = \boxed{12 } \\ $ If the discriminant is positive and not a perfect square like 12, then there are 2 real irrational solutions. $ b^2 - 4ac = 0 $ $ \text{Example :} \\ y = \red 4 x^2 \blue{-28}x + \color{green} {49} \\ \text{Discriminant} \\ \blue{-28}^2 - 4 \cdot \red 4 \cdot \color{green} 49 \\ = \boxed{0} \\ $ $ b^2 - 4ac < 0 $ $ \text{Example :} \\ y = \red x^2 \blue{-3}x + \color{green} 4 \\ \text{Discriminant} \\ \blue{-3}^2 - 4 \cdot \red 1 \cdot \color{green} 4 \\ = \boxed{-7} \\ $ There are only imaginary Solutions. This means that the graph of the quadratic never intersects the axes. Quadratic Equation: $$ y = x^2 + 2x + 1$$ $ a = \red 1 \\ b = \blue 2 \\ a = \color{green} {1} $ The discriminant for this equation is: $ \text{Discriminant } = \blue b^2 -4 \red a \color{green} c \\ \text{Discriminant } = \blue 2^2 -4 \cdot \red 1 \cdot \color{green} \cdot 1 \\ \text{Discriminant } = 4 -4 \\ \text{Discriminant } = \boxed{0} \\ $ Since the discriminant is zero, there should be 1 real solution to this equation. Below is a picture representing the graph and the one solution of $$ y = x^2 + 2x + 1$$. Calculate the discriminant to determine the number and nature of the solutions of the following quadratic equation: $$y = x^2 − 2x + 1 $$. In this quadratic equation, $$ y =\red 1 x^2 + \blue {-2} + \color {green} 1 $$ $ \text{Equation : } y =\red 1 x^2 + \blue {-2}x + \color {green} 1 \\ a = \red 1 \\ b = \blue{-2} \\ c = \color{green} 1 $ Using our general formula: $$ \text{Discriminant } \\ \begin{aligned} &= \blue b^2 -4 \cdot \red a \cdot \color{green} c \\ &= \blue {-2}^2 -4 \cdot \red 1 \cdot \color{green} 1 \\ &= \boxed{0} \end{aligned} $$ Since the discriminant is zero, we should expect 1 real solution which you can see pictured in the graph below. Use the discriminant to find out the nature and number of solutions: $$y = x^2 − x − 2 $$. In this quadratic equation, $$ y =\red 1 x^2 + \blue {-1}x + \color {green} 1 $$ $ \text{Equation : } y =\red 1 x^2 + \blue {-1} + \color {green} 1 \\ a = \red 1 \\ b = \blue{-1} \\ c = \color{green} {2} $ Using our general formula: $$ \text{Discriminant } \\ \begin{aligned} &= \blue b^2 -4 \cdot \red a \cdot \color{green} c \\ &= \blue {-1}^2 -4 \cdot \red 1 \cdot \color{green} {-2} \\ &= 1 - -8 \\ &= 1 + 8 = \boxed 9 \end{aligned} $$ Since the discriminant is positive and rational, there should be 2 real rational solutions to this equation. As you can see below, if you use the quadratic formula to find the actual solutions, you do indeed get 2 real rational solutions. Calculate the discriminant to determine the nature and number of solutions: $y = x^2 - 1$. In this quadratic equation, $y = 1x^2 - 1$. $$ \color{Red}{b^2} - 4\color{Magenta}{a}\color{Blue}{c} \\ \color{Red}{(0)^2} - 4\color{Magenta}{(1)}\color{Blue}{(-1)} = 4 $$ Since the discriminant is positive and a perfect square, we have two real solutions that are rational. Again if you'd like to see the actual solutions and the graph, just look below: Calculate the discriminant to determine the nature and number of solutions: $y = x^2 + 4x - 5$. In this quadratic equation, $y = x^2 + 4x - 5$. $$ \color{Red}{b^2} - 4\color{Magenta}{a}\color{Blue}{c} \\ \color{Red}{(4)^2} - 4\color{Magenta}{(1)}\color{Blue}{(-5)} \\ 16 - 4(-5) = 16 +20 \\ = 36 $$ Since this quadratic equation's discriminant is positive and a perfect square, there are two real solutions that are rational. Calculate the discriminant to determine the nature and number of solutions: $y = x^2 - 4x + 5$. In this quadratic equation, $y = x^2 - 4x + 5$. $$ \color{Red}{b^2} - 4\color{Magenta}{a}\color{Blue}{c} \\ \color{Red}{(-4)^2} - 4\color{Magenta}{(1)}\color{Blue}{(5)} \\ = 16 - 20 = -4 $$ Since the discriminant is negative, there are no real solutions to this quadratic equation. The only solutions are imaginary. Below is a picture of this quadratic's graph. Find the discriminant to determine the nature and number of solutions: $y = x^2 + 4$. $y = x^2 + 4$ $$ \color{Red}{b^2} - 4\color{Magenta}{a}\color{Blue}{c} \\ \color{Red}{(0)^2} - 4\color{Magenta}{(1)}\color{Blue}{(4)} = -16 $$ Since the discriminant is negative, there are two imaginary solutions to this quadratic equation. The solutions are 2i and -2i. Below is a picture of this equations graph. Find the discriminant to determine the nature and number of solutions: $y = x^2 + 25$. $y = x^2 + 25$ $$ \color{Red}{b^2} - 4\color{Magenta}{a}\color{Blue}{c} \\ \color{Red}{(0)^2} - 4\color{Magenta}{(1)}\color{Blue}{(25)} = -100 $$ Since the discriminant is negative, there are two imaginary solutions to this quadratic equation. The solutions are 5i and -5i. This Page: Discriminant Formula Nature of the Roots The discriminant is widely used in the case of quadratic equations and is used to find the nature of the roots. Though finding a discriminant for any polynomial is not so easy, there are formulas to find the discriminant of quadratic and cubic equations that make our work easier. Let us learn more about the discriminant along with its formulas and let us also understand the relation between the discriminant and the nature of the roots. What is Discriminant in Math? Discriminant of a polynomial in math is a function of the coefficients of the polynomial. It is helpful in determining what type of solutions a polynomial equation has without actually finding them. i.e., it discriminates the solutions of the equation (as equal and unequal; real and nonreal) and hence the name "discriminant". It is usually denoted by Δ or D. The value of the discriminant can be any real number (i.e., either positive, negative, or 0). Discriminant Formula The discriminant (Δ or D) of any polynomial is in terms of its coefficients. Here are the discriminant formulas for a cubic equation and quadratic equation. Let us see how to use these formulas to find the discriminant. How to Find Discriminant? To find the discriminant of a cubic equation or a quadratic equation, we just have to compare the given equation with its standard form and determine the coefficients first. Then we substitute the coefficients in the relevant formula to find the discriminant. Discriminant of a Quadratic Equation The discriminant of a quadratic equation ax2 + bx + c = 0 is in terms of its coefficients a, b, and c. i.e., Do you recall using b2 − 4ac earlier? Yes, it is a part of the quadratic formula: x = \(\dfrac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}\). Here, the expression that is inside the square root of the quadratic formula is called the discriminant of the quadratic equation. The quadratic formula in terms of the discriminant is: x = \(\dfrac{-b \pm \sqrt{D}}{2 a}\). Example: Find the discriminant of the quadratic equation 2x2 - 3x + 8 = 0. Comparing the equation with ax2 + bx + c = 0, we get a = 2, b = -3, and c = 8. So the discriminant is, Δ OR D = b2 − 4ac = (-3)2 - 4(2)(8) = 9 - 64 = -55. Discriminant of Cubic Equation The discriminant of a cubic equation ax3 + bx2 + cx + d = 0 is in terms of a, b, c, and d. i.e., Δ or D = b2c2 − 4ac3 − 4b3d − 27a2d2 + 18abcd Example: Find the discriminant of the cubic equation x3 - 3x + 2 = 0. Comparing the equation with ax3 + bx2 + cx + d = 0, we have a = 1, b = 0, c = -3, and d = 2. So its discriminant is, Δ or D = b2c2 − 4ac3 − 4b3d − 27a2d2 + 18abcd = (0)2(-3)2 − 4(1)(-3)3 − 4(0)3(2) − 27(1)2(2)2 + 18(1)(0)(-3)(2) = 0 + 108 - 0 - 108 + 0 = 0 Discriminant and Nature of the Roots The roots of a quadratic equation ax2 + bx + c = 0 are the values of x that satisfy the equation. They can be found using the quadratic formula: x = \(\dfrac{-b \pm \sqrt{D}}{2 a}\). Though we cannot find the roots by just using the discriminant, we can determine the nature of the roots as follows. If Discriminant is Positive If D > 0, the quadratic equation has two different real roots. This is because, when D > 0, the roots are given by x = \(\dfrac{-b \pm \sqrt{\text { Positive number }}}{2 a}\) and the square root of a positive number always results in a real number. So when the discriminant of a quadratic equation is greater than 0, it has two roots which are distinct and real numbers. If Discriminant is Negative If D < 0, the quadratic equation has two different complex roots. This is because, when D < 0, the roots are given by x = \(\dfrac{-b \pm \sqrt{\text { Negative number }}}{2 a}\) and the square root of a negative number leads to an imaginary number always. For example \(\sqrt{-4}\) = 2i. So when the discriminant of a quadratic equation is less than 0, it has two roots which are distinct and complex numbers (non-real). If Discriminant is Equal to Zero If D = 0, the quadratic equation has two equal real roots. In other words, when D = 0, the quadratic equation has only one real root. This is because, when D = 0, the roots are given by x = \(\dfrac{-b \pm \sqrt{\text { 0 }}}{2 a}\) and the square root of a 0 is 0. Then the equation turns into x = -b/2a which is only one number. So when the discriminant of a quadratic equation is zero, it has only one real root. A root is nothing but the x-coordinate of the x-intercept of the quadratic function. The graph of a quadratic function in each of these 3 cases can be as follows. Important Notes on Discriminant: The discriminant of a quadratic equation ax2 + bx + c = 0 is Δ OR D = b2 − 4ac. A quadratic equation with discriminant D has: (i) two unequal real roots when D > 0 (ii) only one real root when D = 0 (iii) no real roots or two complex roots when D < 0 Related Topics: Example 1: Find the discriminant of the following equation: √3x2 + 10x − 8√3 = 0. Solution: The given quadratic equation is √3x2 + 10x − 8√3 = 0. Comparing this with ax2 + bx + c = 0, we get a = √3, b = 10, and c = -8√3. The quadratic discriminant formula is: D = b2 - 4ac = (10)2 - 4(√3)(-8√3) = 100 + 96 = 196 Answer: The discriminant = 196. Example 2: Determine whether each of the following quadratic equation has two real roots, one real root, or no real roots. (a) 3x2 − 5x − 7 = 0 (b) 2x2 + 3x + 3=0. Solution: (a) By comparing the given equation with ax2 + bx + c = 0, we get a = 3, b = -5, and c = -7. Its discriminant is, D = b2 - 4ac = (-5)2 - 4(3)(-7) = 25 + 84 = 109 So we have got a positive discriminant and hence the given equation has two real roots. (b) By comparing the given equation with ax2 + bx + c = 0, we get a = 2, b = 3, and c = 3. Its discriminant is, D = b2 - 4ac = (3)2 - 4(2)(3) = 9 - 24 = -15 So we have got a negative discriminant and hence the given equation has no real roots. Answer: (a) Two real roots (b) No real roots. Example 3: What is the discriminant of quadratic equation 9z2 − 6b2z − (a4 − b4) = 0. Solution: By comparing the given equation with ax2 + bx + c = 0, we get a = 9, b = -6b2, and c = - (a4 − b4). Its discriminant is, D = b2 - 4ac = (-6b2)2 - 4 (9) [-(a4 − b4)] = 36b4 + 36a4 - 36b4 = 36a4 Answer: The discriminant of the given quadratic equation is 36a4. View Answer > go to slidego to slidego to slide Want to build a strong foundation in Math? Go beyond memorizing formulas and understand the 'why' behind them. Experience Cuemath and get started. Book a Free Trial Class FAQs on Discriminant The discriminant in math is defined for polynomials and it is a function of coefficients of polynomials. It tells the nature of roots or in other words, it discriminates the roots. For example, the discriminant of a quadratic equation is used to find: How many roots it has? Whether the roots are real or non-real? What is Discriminant Formula? There re different discriminant formulas for different polynomials: The discriminant of a quadratic equation ax2 + bx + c = 0 is Δ OR D = b2 − 4ac. The discriminant of a cubic equation ax3 + bx2 + cx + d = 0 is Δ or D = b2c2 − 4ac3 − 4b3d − 27a2d2 + 18abcd. How to Calculate the Discriminant of a Quadratic Equation? To calculate the discriminant of a quadratic equation: Identify a, b, and c by comparing the given equation with ax2 + bx + c = 0. Substitute the values in the discriminant formula D = b2 − 4ac. What if Discriminant = 0? If the discriminant of a quadratic equation ax2 + bx + c = 0 is 0 (i.e., if b2 - 4ac = 0), then the quadratic formula becomes x = -b/2a and hence the quadratic equation has only one real root. What Does Positive Discriminant Tell Us? If the discriminant of a quadratic equation ax2 + bx + c = 0 is positive (i.e., if b2 - 4ac > 0), then the quadratic formula becomes x = (-b ± √(positive number)) / 2a and hence the quadratic equation has only two real and distinct roots. What Does Negative Discriminant Tell Us? If the discriminant of a quadratic equation ax2 + bx + c = 0 is negative (i.e., if b2 - 4ac < 0), then the quadratic formula becomes x = (-b ± √(negative number)) / 2a and hence the quadratic equation has only two complex and distinct roots. What is the Formula for Discriminant of Cubic Equation? A cubic equation is of the form ax3 + bx2 + cx + d = 0 and its discriminant is in terms of its coefficients which is given by the formula D = b2c2 − 4ac3 − 4b3d − 27a2d2 + 18abcd. You must have heard the word 'polynomial' or a 'polynomial equation'. Algebra is obsessed with polynomials. You will find them everywhere. So, what is a polynomial then? Polynomial is just an expression that involves variables and constants with arithmetic operators ('+', '-', '×', '÷') in between. An example of a polynomial is, As you can see that there are two variables, x and y, two coefficients i.e., 3 & 4 and a constant 2. Note that a constant is a number on its own, whereas a coefficient is a number multiplied with a variable. A polynomial can have any number of variables. A polynomial can be as simple as, x, i.e., single variable polynomial with a coefficient of 1. Since polynomials involve variables, they can be written as functions of those variables. For example, we have the following single-variable polynomial (there is a single variable x) You can see that it can also be written as the function of as, Therefore, polynomials can be written as functions and are therefore called the polynomial functions. Though, you will find the polynomial of a single variable represented as in textbooks. The subscript 'n' represents the order of the polynomial . The order/degree of a polynomial is the highest power of the variable in the polynomial expression. As an exercise, find the order of following polynomials. Polynomial equation of order 1 is called a monomial equation e.g., . A polynomial equation of 2nd order is called binomial or quadratic equation e.g., . A polynomial equation of order 3 is called a trinomial equation e.g., and so on. From now on we will talk about polynomial equation of one variable only in the rest of this article. Roots of a polynomial equation You may realize that a polynomial equation (of any degree) can be evaluated at different values of input (i.e., at different values of variable). In this way, we can observe how the output of a polynomial equation varies with respect to the value of the variable. We can also graph a polynomial function. Now that we have understood that a polynomial can be evaluated at different values of the variable, we ask the inverse question i.e., "what are the possible values of the variable for which the polynomial has a certain value?" Most of the times in maths, it is of importance to find for which values of the variable, the polynomial equation hits the value of 0. Interesting fact: Graphically, the number of times a polynomial function hits 0 is less than or equal to the order of the polynomial function. Roots of a polynomial equation are those values of the variable for which the polynomial becomes zero. In other words, the roots of a polynomial equation are the solutions of the equation, The number of roots depends on the degree of the polynomial. In fact, the number of roots of a polynomial equation is equal to the degree of that polynomial equation. For example, the polynomial equation has two roots i.e., . If you plug in in the given equation, you will get 0. Note that the degree of polynomial is 2, same as the number of roots. You may, however, realize that the equation, , has only one root i.e., 0, even though the degree of the polynomial is 2. Mathematicians interpret this as a case of repeating roots. In other words, there are 2 roots of this equation, both equal to 0. Finding roots of complex equations is a topic of research in maths. For polynomial equations, we define a term called Discriminant that helps in finding the roots of the equations. Discriminant It is a special kind of quantity that is evaluated with help of unique mathematical relations between the coefficients and constants of a polynomial equation. The discriminant helps determine different characteristics or features of the roots of a polynomial equation. Sometimes knowing the nature of roots is the only information that we seek. Having awareness of the nature of roots helps in deciding whether to proceed with the solution or not. For instance, if we are looking for the equations (and their solutions) that have a particular property, we can know the nature of roots before even solving the equations and based on that information we can decide whether we would like to invest our time in finding the solutions or not. In this article, we will use the word 'Discriminant' to refer to the discriminant of the quadratic equation only. We can write a general quadratic (2nd order polynomial) equation as, You can think of any example of quadratic equation and will find that indeed this is a general representation of the quadratic equation. Now, we would like to find the roots of the equation, In other words, we would like to find the values of x that satisfy the above equation. You can recall (or if you do not know already, you can find the derivation in literature) that the roots of above equation are given by the famous quadratic formula, It is important to note that represents two roots and . Both and are constants (not variables!). Let us solve an example to see the quadratic formula in action. We apply the quadratic formula to find the roots of the equation . The first step in finding the roots is to compare the given equation with the standard quadratic equation and extract the values of coefficients. After comparing the two equations, we find that, Now, we put the above coefficients in the quadratic formula. The roots of the given equation come out to be, And, Therefore, the two roots of the equation come out to be -1 and -4. It is also important to note that we can construct the original quadratic equation with the information of roots. This is done by representing the roots as two individual first order equations and taking their product. Similarly, Now taking the product, That is how we get the original equation. Exercise: As an exercise, find the roots of the following equations. (Hint: Convert it into standard form first) The students are expected to memorize this quadratic formula as it is extensively used everywhere in maths, physics and engineering fields. You will make use of the quadratic formula frequently while solving different types of equations. Using discriminant to find the nature of the roots of quadratic equation The expression under the square root of the quadratic formula, , is known as the discriminant of the quadratic equation. The discriminant is used to find the nature of the roots. Depending on the relative magnitudes of the coefficient, three cases occur. Let us explore the three cases in detail. Case-I (Discriminant < 0) The first case occurs when the value of discriminant is less than 0 i.e., the discriminant is negative. It can be observed that if the discriminant is negative, the quadratic formula becomes. where, (the positive absolute value of the discriminant) Note that is a positive number. Thus, we have In other words, the roots happen to be complex when discriminant < 0. Also, the roots are distinct and appear as conjugate pairs. The condition can be further simplified as . Now we plot the function (Observe that and , i.e., . The graph presents some interesting facts. It can be seen that the function gets to a minimum value of 1, and the curve does not intersect y = 0 line at a single point. In other words, there is not a single real value of x for which the function is 0 i.e., no real solution exists. This is the reason that we get complex roots for the equation . It should be noted that complex values always appear as complex conjugate pairs. Now you must be able to even tell the nature of roots by looking at the graph of a quadratic equation or, conversely, tell that whether the curve crosses the line y = 0 (i.e., x-axis) by knowing the nature of the roots. Caution: Do not mix up the two expressions: The first expression represents a function that can be plotted to get a graph. It gets different x values as input and returns as output. On the other hand, is an equation. Its purpose is to find the value of x for which is 0. The roots are found for (not just). In other words, to say that "find roots of " makes no sense. Case-II (Discriminant = 0) The second case occurs when the value of discriminant is equal to 0. It can be observed that when the discriminant is 0, the quadratic formula simplifies as, Thus, the roots of the quadratic equation are real and equal when discriminant is 0. In other words, we can say that the roots are real and repeated. The condition can be simplified as . It can be reiterated in other words as the roots of a quadratic equation happen to be real and repeated when . Now we plot a relevant function (Observe that and i.e.,). Finally, we see something real (real roots). Again, useful information can be extracted from such a graph. Notice that the curve crosses the x-axis at only one point i.e., x = -1. Thus, we can conclude that the roots of the equation are real (because of the fact that the curve did touch/cross the x-axis and roots are repeated (because of the fact that the x-axis was crossed at only one point). Case-III (Discriminant > 0) The third case occurs when discriminant > 0 i.e., when the discriminant is positive. In this case, the quadratic equation simplifies as where, (the positive absolute value of the discriminant) Note that is a positive number. Thus, we have In short, the roots are real and distinct when the discriminant is positive. The condition can be further simplified as . Therefore, the roots of a quadratic equation are real and distinct when . It is noted that the curve crosses x-axis at two different locations. In other words, the roots of the equation are real (because the real x-axis is crossed by the curve) and distinct (because x-axis is crossed by the curve at two different values of x). We can conclude the three cases as: (Roots are complex and distinct) (Roots are real and repeated) (Roots are real and distinct) Exercise: Apply the concept of discriminant to find the nature of the roots of the following equations. Bonus Exercise: It is known that the roots of a polynomial equation are real and repeated. Based on this information, find the nature of the roots of the following equations. Sources