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(n factorial notation) then have a look the factorial lessonHow To Use The Combination Formula To Solve Word Problems?Example:In how many ways can a coach choose three swimmers from among five swimmers?Solution:There are 5 swimmers to be taken 3 at a time.Using the formula:The coach can choose the swimmers in 10 ways.Example:Six friends want to play enough games of chess to be sure every one plays everyone else. Howmany games will they have to play?Solution:There are 6 players to be taken 2 at a time.Using the formula:They will need to play 15 games. Example:In a lottery, each ticket has 5 one-digit numbers 0-9 on it. a) You win if your ticket has the digits in any order. What are your changes of winning?b) You would win only if your ticket has the digits in the required order. What are yourchances of winning?Solution:There are 10 digits to be taken 5 at a time.a) Using the formula:The chances of winning are 1 out of 252.b) Since the order matters, we should use permutation instead of combination. P(10, 5) = 10 x 9 x 8 x 7 x 6 = 30240The chances of winning are 1 out of 30240.How To Evaluate Combinations As Well As Solve Counting Problems Using Combinations?A combination is a grouping or subset of items. For a combination, the order does not matter.How many committees of 3 can be formed from a group of 4 students? This is a combination and can be written as C(4,3) or 4C3or \(\left({\begin{array}{*{20}{c}}4\\3\end{array}} \right)\).Examples:The soccer team has 20 players. There are always 11 players on the field. How many differentgroups of players can be on the field at any one time?A student need 8 more classes to complete her degree. If she met the prerequisites for allthe courses, how many ways can she take 4 classes next semester?There are 4 men and 5 women in a small office. The customer wants a site visit from a groupof 2 man and 2 women. How many different groups can be formed from the office?Show Video Lesson Examples:A museum has 7 paintings by Picasso and wants to arrange 3 of them on the same wall. How manyways are there to do this?How many ways can you arrange the letters in the word LOLLIPOP?A person playing poker is dealt 5 cards. How many different hands could the player have beendealt?Show Video LessonHow To Solve Combination Problems That Involve Selecting Groups Based On Conditional Criteria?Example:A bucket contains the following marbles: 4 red, 3 blue, 4 green, and 3 yellow making 14 totalmarbles. Each marble is labeled with a number so they can be distinguished.How many sets/groups of 4 marbles are possible?How many sets/groups of 4 are there such that each one is a different color?How many sets of 4 are there in which at least 2 are red?How many sets of 4 are there in which none are red, but at least one is green?Show Video Lesson Try out our new and fun Fraction Concoction Game.Add and subtract fractions to make exciting fraction concoctions following a recipe.There are four levels of difficulty: Easy, medium, hard and insane. Practice the basicsof fraction addition and subtraction or challenge yourself with the insane level. We welcome your feedback, comments and questions about this site or page. Please submit your feedback or enquiries via our Feedback page. Combination questions with solutions are given here to practice and to understand how and when to use the concept of combinations while solving a problem. Also, try important permutation and combination questions for class 11. In combinatorics, the combination is a way of selecting something from a given collection. For example, we have to form a team of 4 people from the given ten persons. In combination, we can determine different ways of selecting 4 persons from 10 persons. The formula for Combination: To find the number of ways to select r items from n items without repetition, we have the formula for combination as follows: \(\begin{array}{l}^{n}C_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}\end{array} \) Learn more about Permutation and Combination. Video Lesson on Formulas for CombinationCombination Questions with Solution Practice the following combination questions, using the formulas for combination. Question 1: If 18Cr = 18Cr + 2, find rC5. Solution: We know that nCr = nC n r, applying this formula 18Cr = 18Cr + 2 18C18 r = 18Cr + 2 18 r = r + 2 2r = 18 2 r = 16/2 = 8 Then, \(\begin{array}{l}^{8}C_{5}=\frac{8!}{5!(8-5)!}= \frac{8\times7\times6}{3\times 2\times 1}=56\end{array} \) Question 2: If nCr : nCr + 1 = 1 : 2 and nCr + 1 : nCr + 2 = 2 : 3, find the value of n and r. Solution: Given, nCr : nCr + 1 = 1 : 2 \(\begin{array}{l}\Rightarrow \frac{n!}{r!(n-r)!}:\frac{n!}{(r+1)!(n-r-1)!}=1:2\end{array} \) \(\begin{array}{l}\Rightarrow \frac{n!}{r!(n-r)!}\times\frac{(r+1)!(n-r-1)!}{n!}=\frac{1}{2}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{n!}{r!(n-r)(n-r-1)!}\times\frac{(r+1)r!(n-r-1)!}{n!}=\frac{1}{2}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{(r+1)}{(n-r)}=\frac{1}{2}\end{array} \) n 3r 2 = 0 .(i) Also, nCr + 1 : nCr + 2 = 2 : 3 \(\begin{array}{l}\Rightarrow \frac{n!}{(r+1)!(n-r-1)!}:\frac{n!}{(r+2)!(n-r-2)!}=2:3\end{array} \) \(\begin{array}{l}\Rightarrow \frac{n!}{(r+1)!(n-r-1)!}\times\frac{(r+2)!(n-r-2)!}{n!}=\frac{2}{3}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{n!}{(r+1)!(n-r-1)(n-r-2)!}\times\frac{(r+2)(r+1)!(n-r-2)!}{n!}=\frac{2}{3}\end{array} \) \(\begin{array}{l}\Rightarrow \frac{(r+2)}{(n-r-1)}=\frac{2}{3}\end{array} \) 2n 5r 8 = 0 .(ii) Multiplying (i) by 2 on both sides subtracting it from (ii), we get 5r + 6r 8 + 4 = 0 r = 4 and n = 14. Some Important Results: nCr = nC n r nCx = nCy and if x y, then x + y = n nCr + nCr 1 = n + 1Cr (Pascals Rule) nCr 1/n 1Cr 1 = n/(n r + 1) nCr/n 1Cr 1 = n/r nCr/nCr + 1 = (r + 1)/(n r) Question 3: In how many ways 5 students can be chosen from 12 students? Solution: The required number of ways = 12C5 \(\begin{array}{l}={^{12}}C_{7}=\frac{12!}{7!(12-7)!}=\frac{12! }{ 7!\times5!}= 792\end{array} \) Question 4: There are 10 people at a party who shakes hands with each other. If each two of them shake hands with each other, how many handshakes happen at the party? Solution: When two people shake hands it is counted as one handshake. Total number of handshakes = 10C2 = 10!/(2! 8!) = 45. Question 5: How many diagonals are there of a 12-sided polygon? Solution: A diagonal can be formed by joining two non-adjacent vertices. Number of diagonals of a 12 sided polygon = number of line segment in a 12 sided polygon 12 edges of the polygon = 12C2 12 = 12!/(2! 10!) 12 = 66 12 = 54. there are 54 diagonals. Also read: Questions 6: How many ways a 5-member committee can be formed out of 10 people if two particular people must be included? Solution: Number of people to be selected if two particular people must be included = 5 2 = 3 Number of ways of selecting 3 out of (10 2) = 8 people = 8C3 = 8!/(3! 5!) = 56. there are 56 ways of forming such a committee Question 7: How many triangles can be formed using 10 points in a plane out of which 4 are collinear? Solution: Given 10 points on a plane out of which 4 are collinear, then 6 points are non-collinear. Number of triangles formed by using 6 non-collinear points = 6C3 = 20 Number of triangles formed by using 6 non-collinear points and one out of the 4 collinear points = 6C2 4C1 = 15 4 = 60 Number of triangles formed by using 6 non-collinear points and two out of the 4 collinear points = 6C1 4C2 = 6 6 = 36 Total number of triangles can be formed = 20 + 60 + 36 = 116. Question 8: There are 4 balls of colour red, green, yellow and blue. In how many ways 2 two balls can be selected such that one of them is red or blue? Solution: Number of ways two balls can be selected out of 4 balls = 4C2 = 6 ways Number of ways two balls can be selected such that neither red nor blue ball gets selected = (4 2)C2 = 2C2 = 1 Number of ways two balls can be selected such that either a red or a blue ball gets selected = 6 1 = 5 ways. Question 9: A team of four has to be selected from 6 boys and 4 girls. How many different ways a team can be selected if at least one boy must be there in the team? Solution: Combination of a four-member team with at least one boy are: {(BGGG), (BBGG), (BBBG), (BBBB)} Number of ways one boy and three girls can be selected = 6C1 4C3 = 6 4 = 24 Number of ways two boys and two girls can be selected = 6C2 4C2 = 15 6 = 90 Number of ways three boys and one girl can be selected = 6C3 4C1 = 20 4 = 80 Number of ways four boys can be selected = 6C4 = 15 Total number of ways to form such a team = 24 + 90 + 80 + 15 = 209. Question 10: It is compulsory to answer 10 questions in an examination choosing at least 4 questions from each part A and part B. If there are 6 questions in part A and 7 questions in part B, in how many ways can 10 questions be attempted? Solution: Case I: 4 questions from part A and 6 questions from part B Number of ways of choosing 4 questions from part A = 6C4 = 15 Number of ways of choosing 6 questions from part B = 7C6 = 7 Total number of ways = 15 7 = 105 Case II: 5 questions from part A and 5 questions from part B Number of ways of choosing 5 questions from part A = 6C5 = 6 Number of ways of choosing 5 questions from part B = 7C5 = 21 Total number of ways = 6 21 = 126 Case II: 6 questions from part A and 4 questions from part B Number of ways of choosing 6 questions from part A = 6C6 = 1 Number of ways of choosing 4 questions from part B = 7C4 = 35 Total number of ways = 1 35 = 35 Required number of ways = 105 + 126 + 35 = 266. Practice Questions on Combinations 1. If 2nCr = 2nCr + 2, find the value of r. 2. Prove that n.(n 1)C(r 1) = (n r + 1).nC(r 1) for all 1 r n. 3. How many diagonals are there of a 15-sided polygon? 4. How many ways a 6-member committee can be formed out of 12 people if two particular people must not be included? 5. A committee of 5 persons have to be formed out of 3 women and 6 men, such that there should be at most 3 women. How many ways can such a committee be formed? 6. How many sides are there of a convex polygon which have 44 diagonals? Learn about various mathematical concepts in a simple manner with detailed information, along with step-by-step solutions to all questions, only at BYJUS. Download BYJUS The Learning App to get personalized videos. The Combinations Calculator will find the number of possible combinations that can be obtained by taking a sample of items from a larger set. Basically, it shows how many different possible subsets can be made from the larger set. For this calculator, the order of the items chosen in the subset does not matter. Factorial There are n! ways of arranging n distinct objects into an ordered sequence, permutations where n = r. Combination The number of ways to choose a sample of r elements from a set of n distinct objects where order does not matter and replacements are not allowed. Permutation The number of ways to choose a sample of r elements from a set of n distinct objects where order does matter and replacements are not allowed. When n = r this reduces to n!, a simple factorial of n. Combination Replacement The number of ways to choose a sample of r elements from a set of n distinct objects where order does not matter and replacements are allowed. Permutation Replacement The number of ways to choose a sample of r elements from a set of n distinct objects where order does matter and replacements are allowed. n the set or population r subset of n or sample set Combinations Formula: \(C(n,r) = \dfrac{n!}{(r! (n - r)!)} \) For n r 0. The formula show us the number of ways a sample of r elements can be obtained from a larger set of n distinguishable objects where order does not matter and repetitions are not allowed. [1] "The number of ways of picking r unordered outcomes from n possibilities." [2] Also referred to as r-combination or "n choose r" or the binomial coefficient. In some resources the notation uses k instead of r so you may see these referred to as k-combination or "n choose k." Combination Problem 1 Choose 2 Prizes from a Set of 6 Prizes You have won first place in a contest and are allowed to choose 2 prizes from a table that has 6 prizes numbered 1 through 6. How many different combinations of 2 prizes could you possibly choose? In this example, we are taking a subset of 2 prizes (r) from a larger set of 6 prizes (n). Looking at the formula, we must calculate 6 choose 2. C (6,2)= 6!/(2! * (6-2)!) = 6!/(2! * 4!) = 15 Possible Prize Combinations The 15 potential combinations are {1,2}, {1,3}, {1,4}, {1,5}, {1,6}, {2,3}, {2,4}, {2,5}, {2,6}, {3,4}, {3,5}, {3,6}, {4,5}, {4,6}, {5,6} Combination Problem 2 Choose 3 Students from a Class of 25 A teacher is going to choose 3 students from her class to compete in the spelling bee. She wants to figure out how many unique teams of 3 can be created from her class of 25. In this example, we are taking a subset of 3 students (r) from a larger set of 25 students (n). Looking at the formula, we must calculate 25 choose 3. C (25,3)= 25!/(3! * (25-3)!)= 2,300 Possible Teams Combination Problem 3 Choose 4 Menu Items from a Menu of 18 Items A restaurant asks some of its frequent customers to choose their favorite 4 items on the menu. If the menu has 18 items to choose from, how many different answers could the customers give? Here we take a 4 item subset (r) from the larger 18 item menu (n). Therefore, we must simply find 18 choose 4. C (18,4)= 18!/(4! * (18-4)!)= 3,060 Possible Answers Handshake Problem In a group of n people, how many different handshakes are possible? First, let's find the total handshakes that are possible. That is to say, if each person shook hands once with every other person in the group, what is the total number of handshakes that occur? A way of considering this is that each person in the group will make a total of n-1 handshakes.Since there are n people, there would be n times (n-1) total handshakes. In other words, the total number of people multiplied by the number of handshakes that each can make will be the total handshakes. A group of 3 would make a total of 3(3-1) = 3 * 2 = 6. Each person registers 2 handshakes with the other 2 people in the group; 3 * 2. Total Handshakes = n(n-1) However, this includes each handshake twice (1 with 2, 2 with 1, 1 with 3, 3 with 1, 2 with 3 and 3 with 2) and since the orginal question wants to know how many different handshakes are possible we must divide by 2 to get the correct answer. Total Different Handshakes = n(n-1)/2 Handshake Problem as a Combinations Problem We can also solve this Handshake Problem as a combinations problem as C(n,2). n (objects) = number of people in the group r (sample) = 2, the number of people involved in each different handshake The order of the items chosen in the subset does not matter so for a group of 3 it will count 1 with 2, 1 with 3, and 2 with 3 but ignore 2 with 1, 3 with 1, and 3 with 2 because these last 3 are duplicates of the first 3 respectively. \(C(n,r) = \dfrac{n!}{(r! (n - r)!)} \) \(C(n,2) = \dfrac{n!}{(2! (n - 2)!)} \) expanding the factorials, cancelling and simplifying, \(= \dfrac{(n-1)\times(n)}{2} = \dfrac{n(n-1)}{2} \) which is the same as the equation above. Sandwich Combinations Problem This is a classic math problem and asks something like How many sandwich combinations are possible? and this is how it generally goes. Calculate the possible sandwich combinations if you can choose one item from each of the four categories: 1 bread from 8 options 1 meat from 5 options 1 cheese from 5 options 1 topping from 3 options Often you will see the answer, without any reference to the combinations equation C(n,r), as the multiplication of the number possible options in each of the categories. In this case we calculate: 8 5 5 3 = 600 possible sandwich combinations In terms of the combinations equation below, the number of possible options for each category is equal to the number of possible combinations for each category since we are only making 1 selection; for example C(8,1) = 8, C(5,1) = 5 and C(3,1) = 3 using the following equation: C(n,r) = n! / (r!(n - r)!) We can use this combinations equation to calculate a more complex sandwich problem. Sandwich Combinations Problem with Multiple Choices Calculate the possible combinations if you can choose several items from each of the four categories: 1 bread from 8 options 3 meats from 5 options 2 cheeses from 5 options 0 to 3 toppings from 3 options Applying the combinations equation, where order does not matter and replacements are not allowed, we calculate the number of possible combinations in each of the categories. You can use the calculator above to prove that each of these is true. 1 bread from 8 options is C(8,1) = 8 3 meats from 5 options C(5,3) = 10 2 cheeses from 5 options C(5,2) = 10 0 to 3 toppings from 3 options; we must calculate each possible number of choices from 0 to 3 and get C(3,0) + C(3,1) + C(3,2) + C(3,3) = 8 Multiplying the possible combinations for each category we calculate: 8 10 10 8 = 6,400 possible sandwich combinations How many possible combinations are there if your customers are allowed to choose options like the following that still stay within the limits of the total number of portions allowed: 2 portions of one meat and 1 portion of another? 3 portions of one meat only? 2 portions of one cheese only? In the previous calculation, replacements were not allowed; customers had to choose 3 different meats and 2 different cheeses. Now replacements are allowed, customers can choose any item more than once when they select their portions. For meats and cheeses this is now a combinations replacement or multichoose problem using the combinations with replacements equation: CR(n,r) = C(n+r-1, r) = (n+r-1)! / (r! (n - 1)!) For meats, where the number of objects n = 5 and the number of choices r = 3, we can calculate either combinations replacement CR(5,3) = 35 or substitute terms and calculate combinations C(n+r-1, r) = C(5+3-1, 3) = C(7, 3) = 35. Calculating cheese choices in the same way, we now have the total number of possible options for each category at bread is 8 meat is 35 cheese is 15 toppings is 8 and finally we multiply to find the total 8 35 15 8 = 33,600 possible sandwich combinations! How many combinations are possible if customers are also allowed replacements when choosing toppings? References [1] Zwillinger, Daniel (Editor-in-Chief). CRC Standard Mathematical Tables and Formulae, 31st Edition New York, NY: CRC Press, p.206, 2003. For more information on combinations and binomial coefficients please see Wolfram MathWorld: Combination. In English we use the word "combination" loosely, without thinking if the order of things is important. In other words: "My fruit salad is a combination of apples, grapes and bananas" We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", its the same fruit salad. "The combination to the safe is 472". Now we do care about the order. "724" won't work, nor will "247". It has to be exactly 4-7-2. So, in Mathematics we use more precise language: When the order doesn't matter, it is a Combination. When the order does matter it is a Permutation. So, we should really call this a "Permutation Lock"! In other words: A Permutation is an ordered Combination. To help you to remember, think "Permutation ... Position" Permutations There are basically two types of permutation: Repetition is Allowed: such as the lock above. It could be "333". No Repetition: for example the first three people in a running race. You can't be first and second. 1. Permutations with Repetition These are the easiest to calculate. When a thing has n different types ... we have n choices each time! For example: choosing 3 of those things, the permutations are: n n n (n multiplied 3 times) More generally: choosing r of something that has n different types, the permutations are: n n ... (r times) (In other words, there are n possibilites for the first choice, THEN there are n possibilites for the second choice, and so on, multplying each time.) Which is easier to write down using an exponent of r: n n ... (r times) = nr Example: in the lock above, there are 10 numbers to choose from (0,1,2,3,4,5,6,7,8,9) and we choose 3 of them: 10 10 ... (3 times) = 103 = 1,000 permutations So, the formula is simply: nr where n is the number of things to choose from,and we choose r of them,repetition is allowed,and order matters. 2. Permutations without Repetition In this case, we have to reduce the number of available choices each time. Example: what order could 16 pool balls be in? After choosing, say, number "14" we can't choose it again. So, our first choice has 16 possibilites, and our next choice has 15 possibilities, then 14, 13, 12, 11, ... etc. And the total permutations are: 16 15 14 13 ... = 20,922,789,888,000 But maybe we don't want to choose them all, just 3 of them, and that is then: 16 15 14 = 3,360 In other words, there are 3,360 different ways that 3 pool balls could be arranged out of 16 balls. Without repetition our choices get reduced each time. But how do we write that mathematically? Answer: we use the "factorial function" ! The factorial function (symbol: !) just means to multiply a series of descending natural numbers. Examples: 4! = 4 3 2 1 = 24 7! = 7 6 5 4 3 2 1 = 5,040 1! = 1 Note: it is generally agreed that 0! = 1. It may seem funny that multiplying no numbers together gets us 1, but it helps simplify a lot of equations. So, when we want to select all of the billiard balls the permutations are: 16! = 20,922,789,888,000 But when we want to select just 3 we don't want to multiply after 14. How do we do that? There is a neat trick: we divide by 13! 16 15 14 13 12 ...13 12 ... = 16 15 14 That was neat: the 13 12 ... etc gets "cancelled out", leaving only 16 15 14. The formula is written: n!(n r)! where n is the number of things to choose from,and we choose r of them,no repetitions,order matters. 16!(163)! = 16!13! = 20,922,789,888,0006,227,020,800 = 3,360 (which is just the same as: 16 15 14 = 3,360) 10!(102)! = 10!8! = 3,628,80040,320 = 90 (which is just the same as: 10 9 = 90) Notation Instead of writing the whole formula, people use different notations such as these: P(n,r) = nPr = nPr = n!(nr)! Examples: P(10,2) = 90 10P2 = 90 10P2 = 90 There are also two types of combinations (remember the order does not matter now): Repetition is Allowed: such as coins in your pocket (5,5,5,10,10) No Repetition: such as lottery numbers (2,14,15,27,30,33) 1. Combinations with Repetition Actually, these are the hardest to explain, so we will come back to this later. 2. Combinations without Repetition This is how lotteries work. The numbers are drawn one at a time, and if we have the lucky numbers (no matter what order) we win! The easiest way to explain it is to: assume that the order does matter (ie permutations), then alter it so the order does not matter. Going back to our pool ball example, let's say we just want to know which 3 pool balls are chosen, not the order. We already know that 3 out of 16 gave us 3,360 permutations. But many of those are the same to us now, because we don't care what order! For example, let us say balls 1, 2 and 3 are chosen. These are the possibilites: Order does matter Order doesn't matter 1 2 31 3 22 1 32 3 13 1 23 2 1 1 2 3 So, the permutations have 6 times as many possibilites. In fact there is an easy way to work out how many ways "1 2 3" could be placed in order, and we have already talked about it. The answer is: 3! = 3 2 1 = 6 (Another example: 4 things can be placed in 4! = 4 3 2 1 = 24 different ways, try it for yourself!) So we adjust our permutations formula to reduce it by how many ways the objects could be in order (because we aren't interested in their order any more): n!(nr)! x 1r! = n!r!(nr)! That formula is so important it is often just written in big parentheses like this: n!r!(nr)! = (nr) where n is the number of things to choose from,and we choose r of them,no repetition,order doesn't matter. It is often called "n choose r" (such as "16 choose 3") And is also known as the Binomial Coefficient. Notation All these notations mean "n choose r": C(n,r) = nCr = nCr = (nr) = n!r!(nr)! Just remember the formula: n!r!(n r)! So, our pool ball example (now without order) is: 16!3!(163)! = 16!3! 13! = 20,922,789,888,0006 6,227,020,800 = 560 Notice the formula 16!3! 13! gives the same answer as 16!13! 3! So choosing 3 balls out of 16, or choosing 13 balls out of 16, have the same number of combinations: 16!3!(163)! = 16!13!(1613)! = 16!3! 13! = 560 In fact the formula is nice and symmetrical: n!r!(nr)! = (nr) = (nnr) Also, knowing that 16!/13! reduces to 161514, we can save lots of calculation by doing it this way: 161514321 = 33606 = 560 Pascal's Triangle We can also use Pascal's Triangle to find the values. Go down to row "n" (the top row is 0), and then along "r" places and the value there is our answer. Here is an extract showing row 16: 1 14 91 364 ...1 15 105 455 1365 ...1 16 120 560 1820 4368 ... 1. Combinations with Repetition OK, now we can tackle this one ... Let us say there are five flavors of icecream: banana, chocolate, lemon, strawberry and vanilla. We can have three scoops. How many variations will there be? Let's use letters for the flavors: {b, c, l, s, v}. Example selections include {c, c, c} (3 scoops of chocolate) {b, l, v} (one each of banana, lemon and vanilla) {b, v, v} (one of banana, two of vanilla) (And just to be clear: There are n=5 things to choose from, we choose r=3 of them,order does not matter, and we can repeat!) Now, I can't describe directly to you how to calculate this, but I can show you a special technique that lets you work it out. Think about the ice cream being in boxes, we could say "move past the first box, then take 3 scoops, then move along 3 more boxes to the end" and we will have 3 scoops of chocolate! So it is like we are ordering a robot to get our ice cream, but it doesn't change anything, we still get what we want. We can write this down as (arrow means move, circle means scoop). In fact the three examples above can be written like this: {c, c, c} (3 scoops of chocolate): {b, l, v} (one each of banana, lemon and vanilla): {b, v, v} (one of banana, two of vanilla): So instead of worrying about different flavors, we have a simpler question: "how many different ways can we arrange arrows and circles?" Notice that there are always 3 circles (3 scoops of ice cream) and 4 arrows (we need to move 4 times to go from the 1st to 5th container). So (being general here) there are r + (n1) positions, and we want to choose r of them to have circles. This is like saying "we have r + (n1) pool balls and want to choose r of them". In other words it is now like the pool balls question, but with slightly changed numbers. And we can write it like this: (r + n 1)!r!(n 1)! = (r + n 1r) where n is the number of things to choose from,and we choose r of themrepetition allowed,order doesn't matter. Interestingly, we can look at the arrows instead of the circles, and say "we have r + (n1) positions and want to choose (n1) of them to have arrows", and the answer is the same: (r + n 1)!r!(n 1)! = (r + n 1r) = (r + n 1n 1)So, what about our example, what is the answer? (3+51)!3!(51)! = 7!3!4! = 5040624 = 35 There are 35 ways of having 3 scoops from five flavors of icecream. In Conclusion Phew, that was a lot to absorb, so maybe you could read it again to be sure! But knowing how these formulas work is only half the battle. Figuring out how to interpret a real world situation can be quite hard. But at least you now know the 4 variations of "Order does/does not matter" and "Repeats are/are not allowed": Repeats allowed No Repeats Permutations (order matters): nr n!(n r)! Combinations (order doesn't matter): (r + n 1)!r!(n 1)! n!r!(n r)! 708, 1482, 709, 1483, 747, 1484, 748, 749, 1485, 750 Copyright 2025 Rod Pierce Problem 1 :Find the number of ways in which 4 letterscan be selected from the word ACCOUNTANT.Solution : Problem 2 :A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw?Solution :Number of white balls = 2Number of black balls = 3Number of red balls = 4Number of non black balls = 2 + 4 = 6Number of black balls to be drawnNumber of red balls to be drawnTotal balls to be drawn3C16C233C26C133C36C03Number of ways =(3C16C2) +(3C26C1)+(3C36C0) = (3 15) +(3 6) + (1 1)= 45 + 18 + 1= 64Problem 3 :Find the number of strings of 4 letters that can be formed with the letters of the wordEXAMINATION?Solution :There are 11 letters not all different.They are AA, II, NN, E, X, M, T, O.The following combinations are possible:Case 1 :Number of ways selecting 2 alike, 2 alike =3C2 = 3 waysCase 2 : Number of ways selecting 2 alike,2 different =3C17C2==>3 x 21 ==>63 ways.Case 3 :Number of ways selecting all 4 different =8C4= 70 ways.Total number of combinations = 3 + 63 + 70 = 136 ways.Total number of permutations (1) to (3) = 3(4!/2!2!) + 63(4!/2!) + 704!= 18 + 756 + 1680 = 2454Problem 4 :How many triangles can be formed by joining 15 points on the plane, in which no line joining anythree points?Solution :From the given question, we come to know that any three points are not collinear.By selecting any three points out of 15 points, we draw a triangle.Number of ways to draw a triangle = 15C3 = (151413) / (3 2 1) = 455Problem 5 :A committee of 7 members is to be chosen from 6 artists, 4 singers and 5 writers. In how many ways can this be done if in the committee there must be at least one member from each group and at least 3 artists ?Solution :For the given condition, possible ways to select members for a committee of 7 members.(3A, 3S, 1W) ----> 6C34C35C1 = 2045 = 400(3A, 1S, 3W) ----> 6C34C13C1 = 20410 = 800(3A, 2S, 2W) ----> 6C34C25C2 = 20610 = 1200(4A, 2S, 1W) ----> 6C44C25C1 = 1565 = 450(4A, 1S, 2W) ----> 6C44C15C2 = 15410 = 600(5A, 1S, 1W) ----> 6C54C15C1 = 645 = 120Thus, the total no. of ways is= 400 + 800 + 1200 + 450 + 600 + 120= 3570Problem 6 :The supreme court has given a 6 to 3 decisions upholding a lower court. Find the number of ways it can give a majority decision reversing the lower court.Solution :Upholding a lower court means, supporting it for its decision.Reversing a lower court means, opposing it for its decision.In total of 9 cases (6 + 3 = 9), it may give 5 or 6 or 7 or 8 or 9 decisions reversing the lower court. And it can not be 4 or less than 4. Because majority of 9 is 5 or more.The possible combinations in which it can give a majority decision reversing the lower court are5 out of 9 ----> 9C5 = 1266 out of 9 ----> 9C6 = 847 out of 9 ----> 9C7 = 368 out of 9 ----> 9C8 = 99 out of 9 ----> 9C9 = 1Thus, the total number of ways is= 126 + 84 + 36 + 9 + 1= 256Problem 7 :Five bulbs of which three are defective are to be tried in two bulb points in a dark room. Find the number of trials in which the room can be lighted.Solution :Given :3 bulbs are defective out of 5.There are two bulb points in the dark room.One bulb (or two bulbs) in good condition is enough to light the room.Since there are two bulb points, we have to select 2 out of 5 bulbs.No. of ways of selecting 2 bulbs out of 5 is= 5P2= 10(It includes selecting two good bulbs, two defective bulbs, one good bulb and one defective bulb. So, in these 10 ways, room may be lighted or may not be lighted)Number of ways of selecting 2 defective bulbs out of 3 is= 3C2= 3(It includes selecting only two defective bulbs. So, in these 3 ways, room can not be lighted)The number of ways in which the room can be lighted is= 10 - 3= 7 Kindly mail your feedback tov4formath@gmail.comWe always appreciate your feedback.All rights reserved. onlinemath4all.com Here are some carefully chosen combination word problems that will show you how to solve word problems involving combinations.Use the combination formula shown below when the order does not matter The combination word problems will show you how to do the followings:Use the combination formulaUse the multiplication principle and the combination formulaUse the addition principle and the combination formulaUse the multiplication principle, addition principle, and combination formulaWord problem #1There are 18 students in a classroom. How many different eleven-person students can be chosen to play in a soccer team?SolutionThe order in which students are listed once the students are chosen does not distinguish one student from another. You need the number of combinations of 18 potential students chosen 11 at a time.Evaluate nCr with n = 18 and r = 11 18C11 =18!/11!(18 - 11)!18C11 =18171615141312111/11!(765432)18C11 =18171615141312/(765432) 18C11 = 31824There are 31824 different eleven-person students that can be chosen from a group of 18 students.Word problem #2For your biology report, you can choose to write about three of a list of four different animals. Find the number of combinations possible for your report.SolutionThe order in which you write about these 3 animals does not matter as long as you write about 3 animals.EvaluatenCrwith n = 4 and r = 3 There are 4 different ways you can choose 3 animals from a list of 4.Word problem #3A math teacher would like to test the usefulness of a new math game on 4 of the 10 students in the classroom. How many different ways can the teacher pick students?SolutionThe order in which the teacher picks students does not matter.EvaluatenCrwith n = 10 and r = 4 10C4 =109876!/432(6)! There are 210 ways the teacher can pick studentsMore challenging combination word problemsThese combination word problemswill also show you how to use the multiplication principle and the addition principle.Word problem #4A company has 20 male employees and 30 female employees. A grievance committee is to be established. If the committee will have 3 male employees and 2 female employees, how many ways can the committee be chosen?SolutionThis problem has the following two tasks:Task 1: choose 3 males from 20 male employeesTask 2: choose 2 females from 30 female employeesWe need to use the fundamental counting principle, also called the multiplication principle, since we have more than 1 task.Fundamental counting principleIf you have n choices for a first task and m choices for a second task, you have n m choices for both tasks.Therefore, evaluate 20C3 and 30C2 and then multiply20C3 by 30C2 20C3 =20191817!/32(17)! 20C3 =302928!/21(28)! 20C330C2 = 1140 435 = 495900The number of ways the committee can be chosen is 495900Word problem #5Eight candidates are competing to get a job at a prestigious company. The company has the freedom to choose as many as two candidates. In how many ways can the company choose two or fewer candidates.SolutionThe company can choose 2 people, 1 person, or none.Notice that this time we need to use the addition principle as opposed to using the multiplication principle.What is the difference? The key difference here is that the company will choose either 2, 1, or none. The company will not choose 2 people and 1 person at the same time. This does not make sense!Addition principleLet A and B be two events that cannot happen together. If n is the number of choices for A and m is the number of choices for B, then n + m is the number of choices for A and B.Therefore you need to evaluate8C2,8C1, and8C0 and then add8C2,8C1, and8C0 together. Useful shortcuts to find combinationsnC1 = n and nC0 = 1Therefore,10C1= 10 and10C0= 18C2 + 8C1 + 8C0 = 28 + 10 + 1 = 39The company has 39 ways to choose two or fewer candidates.Word problem #6A company has 20 male employees and 30 female employees. A grievance committee is to be established. If the committee will have as many as 3 male employees and as many as 2 female employees, how many ways can the committee be chosen?SolutionThe expression as many asmakes the problem quite complex now since we now have all the following cases to consider.Choose 3 males, 2 males, 1 male, or 0 maleChoose 2 females, 1 female, or 0 female.Here is a complete list of all the different cases.3 males and 2 females3 males and 1 female3 males and 0 female2 males and 2 females2 males and 1 female2 males and 0 female1 male and 2 females1 male and 1 female1 male and 0 female0 male and 2 females0 male and 1 female0 male and 0 femaleWe only need to find20C2 20C2 =201918!/2(18)!3 males and 2 females:20C330C2= 1140 435 = 495900 (done in problem #4)3 males and 1 female:20C330C1 = 1140 30 = 342003 males and 0 female:20C330C0 = 1140 1 = 11402 males and 2 females:20C2 30C2 = 190 435 = 826502 males and 1 female:20C230C1 = 190 30 = 57002 males and 0 female:20C2 30C0 = 190 1 = 1901 male and 2 females:20C1 30C2 = 20 435 = 87001 male and 1 female:20C130C1 = 20 30 = 6001 male and 0 female:20C130C0= 20 1 = 200 male and 2 females:20C0 30C2 = 1 435 = 4350 male and 1 female:20C030C1 = 1 30 = 300 male and 0 female: 20C0 30C0 = 1 1 = 1Add everything: 495900 + 34200 + 1140 + 82650 + 5700 + 190 + 8700 + 600 + 20 + 435 + 30 + 1 = 629566.The number of ways to choose the committee is629566Combination word problemsHow to find the number of combinations 1. If nC12 =nC9 find 21Cn. Solution2. If 15C2r1 = 15C2r+4, find r. Solution3. If nPr = 720, and nCr = 120, find n, r. Solution4. Prove that 15C3 + 2 15C4 + 15C5 = 17C5. Solution5. Prove that 35C5 +4r=0(39r) C4=40C5. Solution6. If (n+1)C8 :(n3) P4 = 57 : 16, find the value of n. Solution7. Prove that 2nCn = [2n 1 3 (2n 1)] / n!. Solution8. Prove that if 1 r n then n (n1) Cr1 = (n r + 1)nCr1.Solution9. (i) A Kabaddi coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court? Solution(ii) There are 15 persons in a party and if each 2 of them shakes hands with each other, how many handshakes happen in the party? Solution(iii) How many chords can be drawn through 20 points on a circle? Solution(iv) In a parking lot one hundred , one year old cars, are parked. Out of them five are to be chosen at random for to check its pollution devices. How many different set of five cars can be chosen? Solution(v) How many ways can a team of 3 boys,2 girls and 1 transgender be selected from 5 boys, 4 girls and 2 transgenders? Solution10. Find the total number of subsets of a set with[Hint: nC0 + nC1 + nC2 + + nCn = 2n](i) 4 elements (ii) 5 elements (iii) n elements Solution11. A trust has 25 members.(i) How many ways 3 officers can be selected? Solution(ii) In how many ways can a President, Vice President and a Secretary be selected? Solution12. How many ways a committee of six persons from 10 persons can be chosen along with a chair person and a secretary? Solution13. How many different selections of 5 books can be made from 12 different books if,(i) Two particular books are always selected? Solution(ii) Two particular books are never selected? Solution14. There are 5 teachers and 20 students. Out of them a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees(i) a particular teacher is included? Solution(ii) a particular student is excluded? Solution15. In an examination a student has to answer 5 questions, out of 9 questions in which 2 are compulsory. In how many ways a student can answer the questions? Solution16. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly three aces in each combination. Solution17. Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans, so that always Indians will be the majority in the committee. Solution18. A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of(i) exactly 3 women?(ii) at least 3 women?(iii) at most 3 women? Solution19. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of mans relative and 3 of the wife s relatives?Solution20. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw? Solution21. Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION?. Solution22. How many triangles can be formed by joining 15 points on the plane, in which no line joining any three points?